Introduction to Mathematics

Statistics and Probability

Exam

9 October 2017

Exam Mark = 10 bonus + marks earned on this exam (between 10 and 100).

1. **Hypothesis testing.** [35 Marks] The Groningen Independence Party (GIP) claims that the majority ("...maybe as much as 90%...", they quote) of the people in Groningen supports independence of the province from the Netherlands.

   (a) [25 Marks] The GIP has performed a random survey of 10 inhabitants that revealed that 9 supported independence. Is this strong enough evidence to claim that the majority of Groningers support independence at a 5% significance level? (Define the two-sided hypotheses, define the test-statistic, calculate the p-value, make a decision and give a conclusion)

   (b) [10 Marks] Further inquiry by a journalist reveals that the GIP has already held 4 surveys with 10 respondents before. All these 4 surveys resulted in fewer than 9 people supporting independence. Recalculate the p-value in the light of this new information. What is your conclusion now, based on the same 5% significance level?

2. **Bayes theorem.** [20 marks] An entomologist spots what might be a rare subspecies of beetle, due to a particular pattern on its back. In the rare subspecies, 98% have this pattern. In the common subspecies, only 5% have this pattern. The rare subspecies accounts for only 0.1% of the population. How likely is it that the beetle that the entomologist spots with the particular pattern on its back, is of the rare subspecies?

3. **Markov Chains.** [35 Marks] A call centre of an insurance company is interested in modelling the working process of its employees from one minute to the next. They observe that if an employee is waiting for a call, then in 40% of the cases, (s)he will be responding to a call the next minute. Employees that are on the phone will in 40% of the cases conclude their phone call to start to work on a follow-up related to this phone-call. During this period they are not available for receiving a phone call. Employees will finish the paper-work the next minute with probability 0.2, in which case they are again available for receiving a phone call. We assume that this process is a Markov chain.

   (a) [10 Marks] Determine the states of the process and its transition probability matrix.

   (b) [5 Marks] Why is this Markov chain irreducible?

   (c) [10 Marks] Calculate the probability that an employee who is waiting for a phone call now will also be waiting for a phone call after three minutes.

   (d) [10 Marks] Using that this Markov Chain is ergodic and irreducible, calculate what fraction of the time employees in this call centre are on the phone. [Hint: use the limiting Markov distribution theorem, which involves the formula $\pi P = \pi$.]